

This is probably one of the most accurate determinations of difference of longitude hitherto made.

The result arrived at in 1829, by transmission of chronometers, was $23^s.54$.

The President then stated that on May 25 signals were passed (in exactly the same manner) to and from Edinburgh, for the determination of the longitude of Edinburgh Observatory, and, as regarded the galvanic part of the operation, with perfect success. There was, however, something suspicious in the indications of one of the chronometers employed at Edinburgh, which threw doubt on the clock-comparisons, and rendered the apparent determination of difference of longitude doubtful. The following result however was obtained, which is free from the influence of uncertainty in the chronometer-rate; that when a signal is given at Greenwich, by means of a Greenwich battery, the time noted for the signal at Edinburgh is later than that noted at Greenwich by $\frac{1}{17}$ th of a second of time (and *vice versa* if the signal is given at Edinburgh by means of an Edinburgh battery). This difference probably arises from two causes: first, the time actually occupied by the transmission of the galvanic pulse, which, according to the American determinations, would explain less than half of the difference; secondly, the circumstance that the galvanic current when it reaches the distant needle is somewhat less vigorous than when it passes the nearer needle, and the languid movement of the distant needle catches the eye more slowly, and is recorded as occurring at a later time.

In reply to a remark by Mr. R. C. Carrington, on the influence of the length of the transit telescope upon the determination of clock error, the President stated that the length of the Greenwich transit telescope is less than 12 feet, and that at Cambridge 10 feet, and that no sensible error would result from that cause.

On Certain Results of Laplace's Formulæ, expressing the Relation between the Inclinations, Excentricities, and other Elements of the Planetary Orbits. By Dr. Lardner.

In this paper the author shows that the well-known formulæ of Laplace,

$$\sum (m \sqrt{a} e^2) = c \quad (\text{I})$$

$$\sum (m \sqrt{a} \tan^2 i) = c' \quad (\text{II})$$

would not of themselves suffice to assure the stability of the planetary system in the sense in which the term is usually applied. By a very simple process he demonstrates that for anything contained in these theorems to the contrary, the excentricities of all the planets which revolve within the orbit of *Jupiter* may attain any magnitude whatever, that their several orbits may actually become

hyperbolic, and that although the formulæ (II) certainly imposes major limits upon the inclinations, these limits are in some cases so considerable as to be inconsistent with the stability of the planetary system as usually understood.

We shall obtain the numerical value of the constant c by calculating severally the products $m \cdot \sqrt{a} \cdot e^2$ for each of the planets; but in the case of the group of small planets between *Mars* and *Jupiter*, their masses being undetermined, cannot with any arithmetical precision enter into this calculation: it is evident, however, that, from their extreme minuteness, they will produce a very inconsiderable effect upon the value of c . It may be sufficient for the present purpose to suppose the group to be replaced by a single planet, having a mass equal to that of the earth, an excentricity equal to that of *Mars*, and a mean distance $= 2.6$. If this be assumed, and the several products $m \sqrt{a} e^2$ be calculated from the tables of the planetary elements, we shall find the following numerical results:—

	$m \sqrt{a} e^2$.
Mercury	0.00000001298
Venus	10
Earth	79
Mars	400
Planetoids.....	3947
Jupiter	504525
Saturn	277266
Uranus	38197
Neptune	2219
$\Sigma (m \sqrt{a} e^2)$	$= 0.00000827941$

It appears, therefore, that

$$c = 0.00000827941.$$

If it be desired then to ascertain the greatest possible excentricity of each planet which is compatible with this constant value of c , it will only be necessary to suppose that $e = 0$ for all the other planets. That will give

$$e'^2 = \frac{c}{m' \sqrt{a'}},$$

where e' is the excentricity, m' the mass, and a' the mean distance of the planet, whose limiting excentricity is sought.

If, then, we substitute in this formula the numerical value of c determined above and the values of e , m , and a for each of the planets severally, we shall find the major limits of the values of e for the planets, including the imaginary planets taken to represent the group of planetoids.

	Major limit of e .
Mercury	5'192
Venus	1'976
Earth	1'714
Mars	4'240
Planetoids.....	1'350
Jupiter	0'0617
Saturn	0'0970
Uranus	0'2170
Neptune	0'1684

It appears, therefore, that for all the planets included within the orbit of *Jupiter*, the major limits of e are greater than 1, and, therefore, the excentricities are susceptible of all possible values between 0 and 1 and of all values between 1 and the several limiting values given above.

It follows from all this that, consistently with the conditions involved in the formula (I), the orbits of these planets may be ellipses of any degree of excentricity from the circle to the most flattened oval, and that they may be even hyperbolas, for which e would have any values between 1 and the several numbers given in the preceding table.

In the case of the four superior planets, *Jupiter*, *Saturn*, *Uranus*, and *Neptune*, the formula does impose such narrow limits upon their excentricities as must effectually guarantee their stability and the uniformity of their climatological conditions.

The formula

$$\Sigma (m \sqrt{a} \tan^2 i) = c'$$

is subject to precisely the same observations.

By substituting, as before, the values of m , a , and i , for each of the planets taken from the tables of the elements, we shall obtain the values of the products $m \sqrt{a} \tan^2 i$ for the planets severally, and by their addition we shall obtain the numerical value of their constant sum. Thus we obtain :—

$$\Sigma (m \sqrt{a} \tan^2 i) \dots\dots = 0\cdot00000314170$$

To find the limiting value of each inclination it will be sufficient, as before, to suppose all the orbits but the one whose limiting inclination is sought to coincide with the plane of the ecliptic. If, then, m' , a' , and i , express the mass, mean distance, and inclination of the planet whose limiting inclination is sought, we shall have

$$\tan^2 i = \frac{c'}{m' \sqrt{a'}}$$

Now, by substituting in these successively for m' and $\sqrt{a'}$ their values for each of the planets, we shall obtain the limiting values of the inclination, which will be as follows :—

	Limit of i .
Mercury	72 38
Venus	50 36
Earth	46 34
Mars	69 3
Planetoids	39 45
Jupiter	2 11
Saturn	3 25
Uranus	7 37
Neptune.....	5 56

It appears, therefore, that in this case, as in that of the excentricities, the conservative influence of the formula only bears upon the outer group of planets, and that the inclinations of the inner group are liable, so far as they are restricted by this formula, to a range of variation altogether incompatible with all idea of conservation.

[Laplace, in the Memoir in which he first announced the theorems above referred to (*Mém. Acad. des Sciences*, 1784), remarked that the results of his researches on the secular variations of the planetary movements (to which these theorems formed direct corollaries) assured the permanent oscillation of the excentricities and inclinations of the orbits within comparatively narrow limits; and this statement was repeated by him in much more explicit terms in the *Mécanique Céleste* (tome i. liv. ii., tome v. liv. xv.) It is important to bear in mind, however, that, although these theorems do not imply quite so much as has been ascribed to them, this circumstance does not in the slightest degree impugn the researches of geometers on the stability of the planetary system. Lagrange, by a comprehensive investigation of the subject, first determined the actual limits within which the planetary orbits would oscillate from the mutual action of the disturbing bodies (*Mem. Acad. Berlin*, 1782), and the researches of that great geometer have been perfected and extended in recent times by M. Le Verrier. Laplace, in the Memoir already referred to, showed that the expressions for the secular variations of the excentricities and inclinations of the planetary orbits would consist of a series of terms involving the sines and cosines of angles proportional to the time, but would not contain any term involving the time either in an algebraic or an exponential form; and as he supposed the coefficients of the various terms to be small from the nature of the problem, he hence concluded that both the excentricities and the inclinations would perpetually oscillate between narrow limits. These researches were subsequently published in an improved form in the *Mécanique Céleste*, but on neither occasion were the actual limits of the excentricity or the inclination in any individual case determined; nor is any allusion made in the latter work to the remarkable researches of Lagrange on this subject, either in the first volume, wherein the subject of

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the secular variations is formally investigated, or in the fifth volume, containing historical notices of the labours of Newton and his successors on the theory of gravitation (with the exception of perhaps a very obscure allusion to the subject at page 319 of tome v.) It may be stated that the inadequacy of the analysis of Laplace, to establish the stability of the planetary orbits individually, without having recourse to numerical calculation, had been already pointed out by Le Verrier and others (*Comptes Rendus*, 1840, 1^{ier} Semestre, p. 526).—R. G.]

Results of Micrometrical Measures of Double Stars made at Tarn Bank, Cumberland, from 1850.2 to 1853.4. By Isaac Fletcher, Esq., F.R.A.S.

The author commences his paper, by referring the reader to the tenth volume of the *Monthly Notices*, for a description of the instrumental means employed by him in obtaining the micrometrical measures here presented to the Society.

The telescope employed is a 6-foot achromatic refractor of 4.14 inches aperture, mounted equatorially in the English fashion on a polar axis of mahogany, and driven by clock-work. The mounting, in fact, appears to be almost precisely similar to those of the instruments of Mr. Bishop, Lord Wrottesley, and the one which formerly belonged to Admiral Smyth, but which now adorns Dr. Lee's Observatory at Hartwell. The object-glass seems to be of first-rate excellence, as is abundantly shown by the measurement of such stars as ζ *Herculis*, δ *Cygni*, &c., and by the fact that, under favourable circumstances, it bears a power of 600. The measures have all been executed with a parallel wire micrometer, and a deep red illumination of the field of view. The hour-circle is *loose* on the polar axis,—an arrangement which appears to be convenient and advantageous, and Mr. Fletcher strongly recommends its general adoption. The entire instrument is the workmanship of Mr. Cooke of York. The catalogue of observations incorporates about 5200 *individual* measures, which are represented by about 280 *sets* of measures, most of the sets containing eight measures of position and *sixteen* measures of distance. In consequence of the greater liability to error in observing distances, Mr. Fletcher seems to have taken great pains to insure good results by a large accumulation of measures of this element.

Stars of an ascertained binary character seem chiefly to have occupied Mr. Fletcher's attention; and, accordingly, the catalogue contains numerous observations of γ *Virginis*, ξ *Ursæ*, 70 *Ophiuchi*, *Castor*, ζ *Herculis*, and other stars of interest. The catalogue is followed by a synopsis of *final* mean results.

Mr. Fletcher gives notes on some of the more remarkable objects to which his measures have reference, comparing his results with those obtained by other astronomers.

We here insert some of these notes:—